2.4

Side to Side

Horizontal Dilations of Quadratic Functions

LEARNING GOALS

In this lesson, you will:

- Graph quadratic functions through horizontal dilations.
- Identify the effect on a graph by replacing $f(x)$ by $f(Bx)$.
- Write quadratic functions given a graph.

KEY TERMS

- horizontal dilation
- horizontal stretching
- horizontal compression

Smile for the camera! FLASH! One of the most uncomfortable things about taking a picture indoors is using a flash. And if the picture-taker didn't quite get the shot, it is a double dose of bright lights! Flashes routinely do a number on your eyes—so does the sun. Even though your eyes routinely dilate depending on the light conditions, the sudden flash of light can be uncomfortable for most people. Of course, other things can make your eyes dilate. For example, if you are hungry and you see a commercial for a tantalizing meal, your eyes will involuntarily dilate. You may also know that your pupils dilate when you are sleeping.

What are other things that might cause your eyes to dilate? Have you ever wondered why a typical eye exam usually includes an eye dilation exam?

PROBLEM 1 Horizontal Stretching and Compressing

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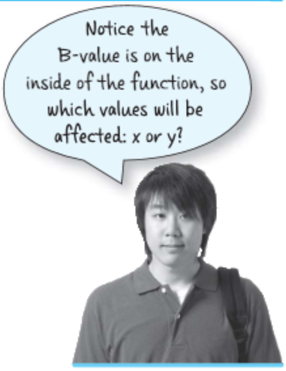
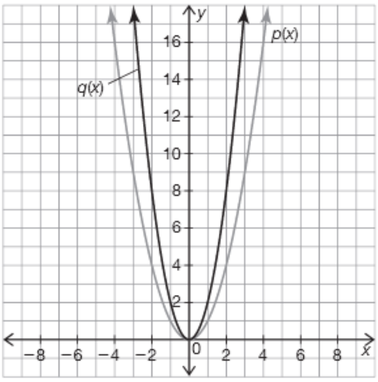
Now, let's explore the effect of the *B*- value in the transformational function $g(x) = Af(B(x - C)) + D$. The constant *B* is a multiplier.

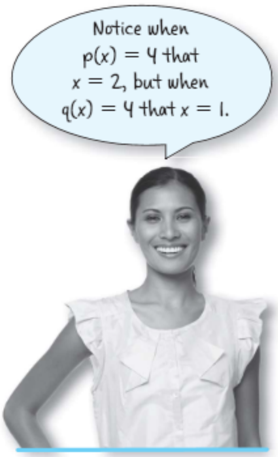


1. Compare the graph of $p(x) = x^2$ with $q(x) = (2x)^2$.



a. Analyze the table of values that correspond to the graph.

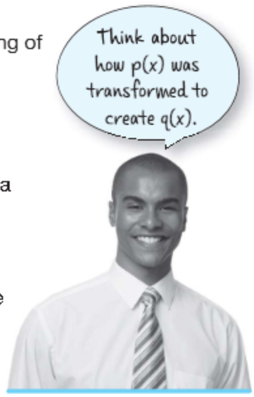
x	p(x)	q(x)
0	0	0
1	1	4
2	4	16
3	9	36
4	16	64
5	25	100
6	36	144



Circle other instances where the *y*-values for each function are the same. Then, list all the points where $p(x)$ and $q(x)$ have the same *y*-value.

b. How do the *x*-values compare when the *y*-values are the same?

A **horizontal dilation** is a type of transformation that stretches or compresses the entire graph. **Horizontal stretching** is the stretching of a graph away from the *y*-axis. **Horizontal compression** is the squeezing of a graph towards the *y*-axis.



c. Complete the statement.

The function $q(x)$ is a ______________________ of $p(x)$ by a factor of ________.

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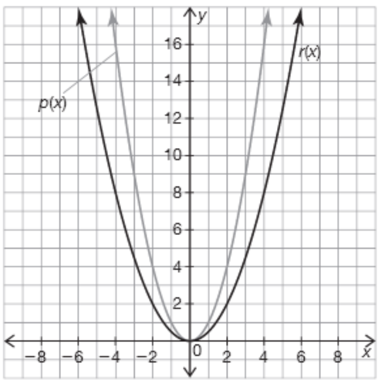
d. How does the factor of stretching or compression compare to the *B*-value in $q(x)$?

2. Now, let's compare the graph of $p(x) = x^2$ with $r(x) = p\left(\frac{1}{2}x\right)$.



a. Analyze the table of values that correspond to the graph.

x	*p(x)*	*r(x)*
0	0	0
1	1	0.25
2	4	1
3	9	2.25
4	16	4
5	25	6.25
6	36	9

Circle instances where the *y*-values for each function are the same. Then, list all the points where $p(x)$ and $r(x)$ have the same *y*-value. The first instance has been circled for you.

b. How do the x-values compare when the y-values are the same?

c. Complete the statement.

The function $r(x)$ is a ______________________ of $p(x)$ by a factor of _________.

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d. How does the factor of stretching or compression compare to the B-value in $q(x)$?

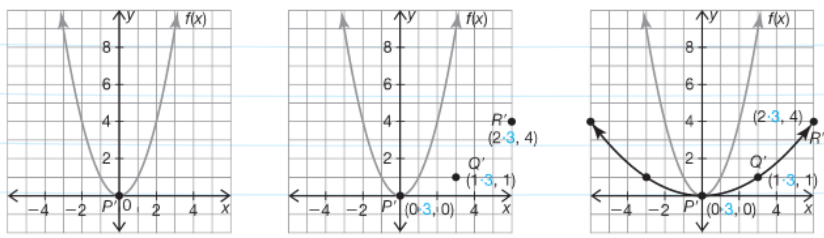
Compared with the graph of $y = f(x)$, the graph of $y = f(Bx)$ is:

- horizontally compressed by a factor of $\frac{1}{|B|}$ if $|B| > 1$.
- horizontally stretched by a factor of $\frac{1}{|B|}$ if $0 < |B| < 1$.

You can use reference points to graph the function $d(x) = f\left(\frac{1}{3}x\right)$ when $f(x) = x^2$.

From $d(x)$ you know that $C = 0$, $D = 0$, and $B = \frac{1}{3}$. The vertex for $d(x)$ is (0, 0).

Notice $0 < |B| < 1$ so the graph will horizontally stretch by a factor of $\frac{1}{\frac{1}{3}}$ or 3.



The function $f(x)$ is shown. First plot the new vertex (C, D). This point establishes the new set of axes.

Next, think about B. To plot Q' move right $1 \cdot 3$ units and up 1 unit from the vertex because all x-coordinates are being stretched by a factor of 3. To plot R' start at the vertex and move to the right $2 \cdot 3$ units and go up 4 units.

Finally, use symmetry to complete the graph.

3. List the corresponding points on $d(x)$ for the given points on $f(x)$ in the worked example.

$f(x)$	$d(x)$
(x, y)	
$(-3, 9)$	
$(-2, 4)$	
$(-1, 1)$	
$(0, 0)$	
$(1, 1)$	
$(2, 4)$	
$(3, 9)$	
$(4, 16)$	

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4. In the worked example, you analyzed $d(x) = f\left(\frac{1}{3}x\right)$ when $f(x) = x^2$.

a. If you were asked to graph $h(x) = f(3x)$, describe how the graph would change.

b. List the corresponding points on $h(x)$ for the given points on $f(x)$.

$f(x)$	$h(x)$
(x, y)	
$(0, 0)$	
$(1, 1)$	
$(2, 4)$	
$(3, 9)$	

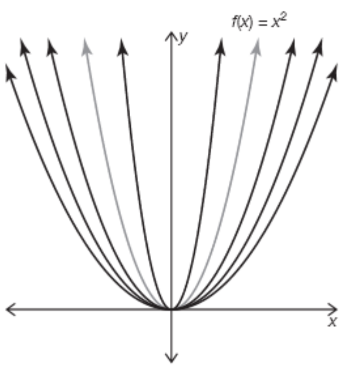
5. Analyze the graphs shown on the coordinate plane. Label each graph shown with its corresponding function.

a. $m(x) = \left(\frac{1}{4}x\right)^2$

b. $v(x) = (3x)^2$

c. $p(x) = \left(\frac{1}{3}x\right)^2$

d. $g(x) = \left(\frac{1}{2}x\right)^2$



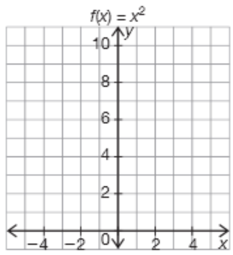
Now that you have studied how $B > 0$ affects the graph of a quadratic function, you will investigate what happens when $B < 0$.

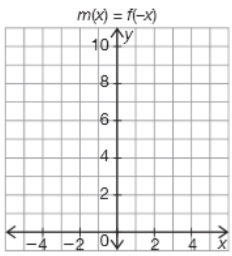
6. Based on what you know about how $A < 0$ values affect the graph of x^2, make a conjecture about how you think $B < 0$ values will affect the graph of x^2.

7. List the corresponding points on $m(x)$ for the given points on $f(x)$.

$f(x) = x^2$		$m(x) = f(-x)$	
(x, y)		$(-x, y)$	
A	$(-3, 9)$	A'	
B	$(-2, 4)$	B'	
C	$(-1, 1)$	C'	
D	$(0, 0)$	D'	
E	$(1, 1)$	E'	
F	$(2, 4)$	F'	
G	$(3, 9)$	G'	

8. Plot and label the points of $f(x)$ and $m(x)$ on the two coordinate planes.





9. Analyze the graphs in Question 8.

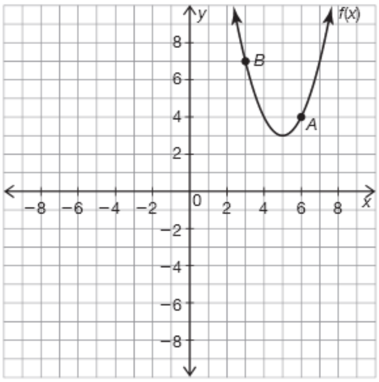
a. What do you notice about the graphs?

b. What do you notice about the corresponding points in $f(x)$ and $m(x)$?

c. Did a transformation occur?

10. Given $f(x) = (x - 5)^2 + 3$.

a. Sketch $d(x) = f(-x)$. Then label A' and B' on your sketch.



b. Write the function for $d(x)$ by substituting $(-x)$ into $f(x)$. Show all your work.

Compared with the graph of $y = f(x)$, the graph of $y = f(Bx)$ is:

- compressed horizontally by a factor of $\frac{1}{|B|}$ if $|B| > 1$.
- stretched horizontally by a factor of $\frac{1}{|B|}$ if $0 < |B| < 1$.
- reflected across the y-axis if $B < 0$.

11. Given $y = f(x)$, write the coordinate notation represented in $y = Af(B(x - C)) + D$.

$(x, y) \rightarrow$ ____________________

12. Connor and Jocelyn each describe the effects of the graph of $d(x) = f(3x + 12)$ when $f(x) = x^2$.

Connor

$d(x) = f(3x + 12)$

$d(x) = f(3(x + 4))$

The B-value is 3 so the graph will have a horizontal compression of $\frac{1}{3}$. The C value is -4, so the vertex will be shifted 4 units to the left at $(-4, 0)$.

Jocelyn

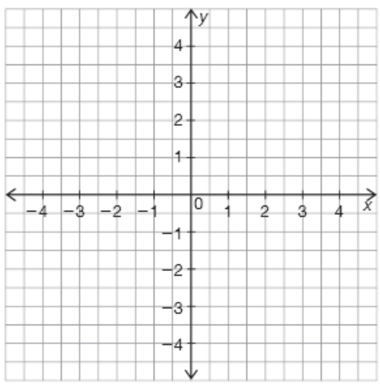
$d(x) = f(3x + 12)$

The B-value is 3 so the graph will have a horizontal compression of $\frac{1}{3}$. The C-value is -12, so the vertex will be shifted 12 units to the left at $(-12, 0)$.

a. Explain how Jocelyn incorrectly described the graph of $d(x)$.

b. Use transformations to sketch the graph of $d(x) = f(3(x + 4))$.

f(*x*)	*d*(*x*)
(0, 0)	
(1, 1)	
(2, 4)	



c. Rewrite $d(x)$ in terms of x.

Connor

I can rewrite the function $d(x)$ in terms of x in different ways.

$d(x) = (3x + 12)^2$ or

$d(x) = (3(x + 4))^2$ or

$d(x) = 9(x + 4)^2$

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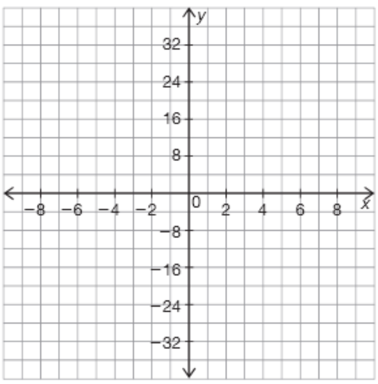
Explain Connor's reasoning.

d. Use transformations to sketch the graph of $d(x) = 9(x + 4)^2$.

f(*x*)	*d*(*x*)
(0, 0)	
(1, 1)	
(2, 4)	



e. Use a graphing calculator to verify that your sketch for $d(x) = f(3(x + 4))$ given $f(x) = x^2$ and $d(x) = 9(x + 4)^2$ is the same.

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f. Why does it make sense that a quadratic function written in transformation notation with a B-value of 3 would produce the same graph as a quadratic function with an A-value of 9?

Talk the Talk

1. Complete the table to describe the graph of each function as a transformation on $y = f(x)$.

Function Form	Equation Information	Description of Transformation of Graph
$y = f(x) + D$	$D > 0$	
	$D < 0$	
$y = f(x - C)$	$C > 0$	
	$C < 0$	
$y = Af(x)$	$\lvert A\rvert > 1$	
	$0 < \lvert A\rvert < 1$	
	$A < 0$	
$y = f(Bx)$	$\lvert B\rvert > 1$	
	$0 < \lvert B\rvert < 1$	
	$B < 0$	

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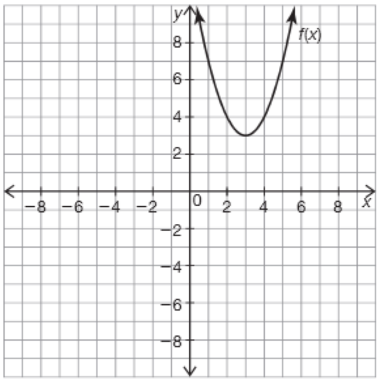
2. Given $y = f(x)$, sketch $m(x) = -f(x)$. Describe the transformations you performed.



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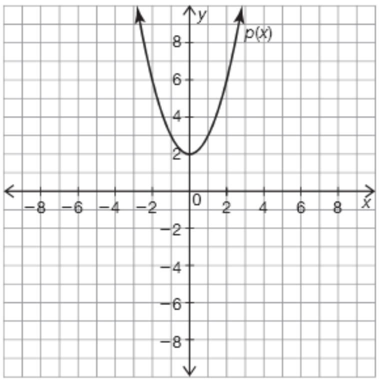
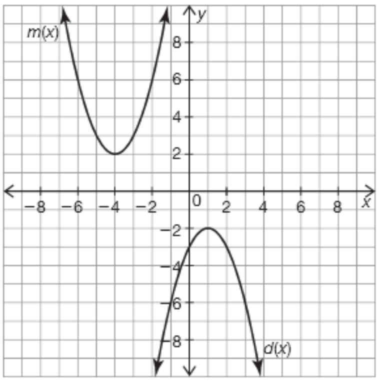
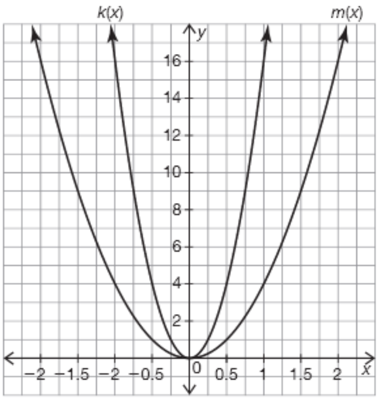
3. Given $y = p(x)$, sketch $m(x) = -p(x + 3)$. Describe the transformations you performed.



4. Write $m(x)$ in terms of $d(x)$.



5. Write $m(x)$ in terms of $k(x)$.



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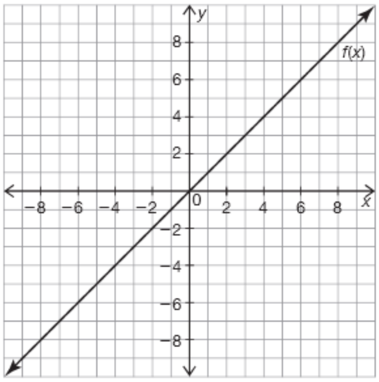
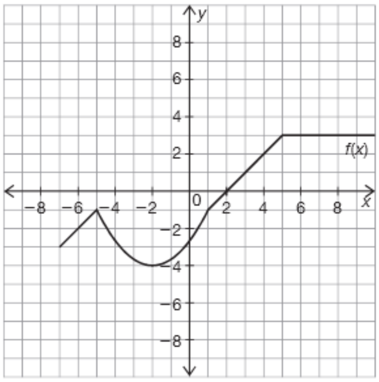
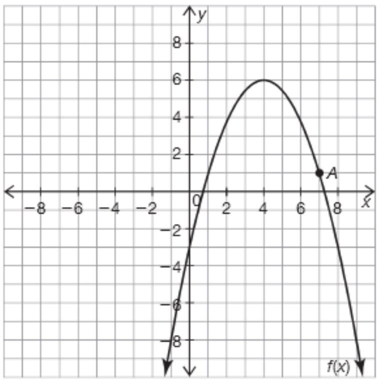
6. Given the graph of $f(x)$, sketch $g(x) = 3f(x + 1) - 6$ on the coordinate plane shown.



7. Given the graph of $f(x)$, sketch $g(x) = f(x - 2) + 3$ on the coordinate plane shown.



8. Given the graph of $f(x)$, sketch $m(x) = f(-x)$ on the coordinate plane shown. Label A'.



Be prepared to share your solutions and methods.